Nanyang Technological University

SPMS/Division of Mathematical Sciences

2015/16 Semester 1 MH1810 Mathematics I Answers for Tutorial 1

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1. (a) $a = 0, b = 0$
   (b) $a = -\frac{1}{5}, b = \frac{2}{5}$

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2. (a) $|z| = 2, \arg(z) = \frac{\pi}{3}$
   (b) $|z| = 2, \arg(z) = \frac{2\pi}{3}$
   (c) $|z| = 2, \arg(z) = -\frac{\pi}{3}$
   (d) $|z| = 2, \arg(z) = -\frac{2\pi}{3}$

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3. (a) $-2i$
   (b) 2
   (c) $1 - 3i$
   (d) $-3 + 4i$
   (e) $2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) = 1 - \sqrt{3}i$

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5. $x = \frac{2}{5}$ $y = -\frac{1}{5}$.

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7. (a) $4e^{\frac{2\pi i}{3}}$
   (b) $e^{\frac{\pi i}{2}}$

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9. (a) $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$
   (b) $\cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$
   (c) $\cos\frac{-\pi}{2} + i\sin\frac{-\pi}{2} = -i$

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10. 8192

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11. $z_0 = 2\text{cis}(\frac{-\pi}{8}),\quad z_1 = 2\text{cis}(\frac{3\pi}{8}),\quad z_2 = 2\text{cis}(\frac{7\pi}{8}),\quad z_3 = 2\text{cis}(\frac{11\pi}{8}) = 2\text{cis}(\frac{-5\pi}{8})$

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12. (a) $z_0 = 2e^{\frac{\pi}{3}} = 1 + \sqrt{3}i, z_1 = 2e^{\frac{4\pi}{3}} = 2e^{\frac{-2\pi}{3}} = -1 - \sqrt{3}i,$
$z_0' = 2e^{\frac{-\pi}{3}} = 1 - \sqrt{3}i,$ and $z_1' = 2e^{\frac{2\pi}{3}} = -1 + \sqrt{3}i.$

(b) $z_0 = e^{\frac{\pi}{4}i} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, z_1 = e^{\frac{3\pi}{4}i} = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i,$
$z_2 = e^{\frac{5\pi}{4}i} = e^{\frac{-3\pi}{4}i} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$ or $z_3 = e^{\frac{7\pi}{4}i} = e^{\frac{-\pi}{4}i} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i.$

(c) $z = -1$ or $z = e^{\frac{\pi}{2}i} = i$ or $z = e^{\frac{3\pi}{2}i} = -i$

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13. $z = 2 + 4i,$ or $z = -1 + i(4 + \sqrt{3})$ or $, z = -1 + i(4 - \sqrt{3}).$

14. $(1 + \alpha^4)(1 + \alpha^3)(1 + \alpha^2)(1 + \alpha) = 1.$

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16. (b) $5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ (c) $\dfrac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$

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