Nanyang Technological University
SPMS/Division of Mathematical Sciences

2015/16 Semester 1 **MH1810 Mathematics I** **Tutorial 5**

Please be remined that there will be a 15-minute quiz during the tutorial session.

Reference: Thomas' Calculus: Chapter 2, Section 2.1 - 2.2, 2.4 - 2.6.

1. Suppose that $\lim_{x\to 1} p(x) = 4$, $\lim_{x\to 1} q(x) = \pi$ and $\lim_{x\to 1} r(x) = 3$. Determine each of the following limits and justify each step by indicating the appropriate Limit Law(s).

   (a) $\lim_{x\to 1} [\pi p(x) + q(x) - (qr)(x)]$ (b) $\lim_{x\to 1} \dfrac{p(x) + q(x)}{r(x)}$

2. Find the limit.

   (a) $\lim_{x\to \pi/2} \cos x$ (b) $\lim_{x\to \infty} 179$ (c) $\lim_{x\to 3^-} (x^2 + \pi x + \sqrt{2})$ (d) $\lim_{y\to 3} 4^y$ (e) $\lim_{t\to 125} \sqrt[3]{t}$

3. Use continuity to determine the following limits.

   (a) $\lim_{x\to 1} \sqrt{\dfrac{x}{1+3x}}$

   (b) $\lim_{x\to 1} \sin(x-1)^2$

   (c) $\lim_{x\to 1} \tan\left(\dfrac{(2-x^2)\pi}{3}\right)$

   (d) $\lim_{x\to 3} \ln|x-2|$

   (e) $\lim_{x\to \sqrt{2}} \tan^{-1}\left(\dfrac{x^2}{2}\right)$

4. Use appropriate techniques to find the following limits.

   (a) $\lim_{x\to 1} \dfrac{x^3-1}{x^2-1}$

   (b) $\lim_{x\to \sqrt{2}} \dfrac{x^2-2}{x-\sqrt{2}}$

   (c) $\lim_{x\to 0^-} \dfrac{x}{\sqrt{x+1}-1}$

   (d) $\lim_{t\to -3} \dfrac{t^2-9}{2t^2+7t+3}$

   (e) $\lim_{x\to 1} \dfrac{\frac{1}{x}-1}{x-1}$

   (f) $\lim_{t\to \frac{\pi}{4}} \dfrac{\cos 2t}{\cos t - \sin t}$

   (g) $\lim_{h\to 0} \dfrac{(2+h)^3-8}{h}$.

   (h) $\lim_{x\to 7^+} \dfrac{\sqrt{x+2}-3}{x-7}$.

   (i) $\lim_{t\to 0^+} \left(\dfrac{1}{t} - \dfrac{1}{t^2+t}\right)$.

(j) $\lim_{x\to 0}\left(x^4\cos\frac{1}{x}\right)$

5. Determine whether $\lim_{x\to 2} f(x)$ exists where

$$f(x)=\begin{cases}\dfrac{3x-6}{x^2-4} & \text{if } 0<x<2,\\ 0 & \text{if } x=2,\\ \dfrac{x-2}{\sqrt{3-x}-1} & \text{if } 2<x<3.\end{cases}$$

6. If the product $h(x)=f(x)\cdot g(x)$ is continuous at $x=0$, is it always true that $f(x)$ and $g(x)$ must be continuous at $x=0$? Give reasons to your answer.

7. Find real constants c and d that makes g continuous at $x=4$.

$$g(x)=\begin{cases}x^2-c^2 & \text{if } x<4,\\ d & \text{if } x=4,\\ cx+20 & \text{if } x>4.\end{cases}$$

8. Suppose $3x\le f(x)\le x^3+2$ for $0\le x\le 2$.

   (a) What is $f(1)$?

   (b) Use Squeeze Theorem to evaluate $\lim_{x\to 1} f(x)$.

   (c) Is f continuous at $x=1$?

9. Under certain circumstances a rumor spreads according the equation

$$p(t)=\frac{1}{1+ae^{-kt}}$$

   where $p(t)$ is the proportion of the population that knows the rumor at time t and a and k are positive constants.

   (a) Find $\lim_{t\to\infty} p(t)$.

   (b) For $a=10$, $k=0.5$ and t being measured in hours, how long will it take for 80% of the population to hear the rumor?

10. Determine the following infinite limits.

   (a) $\lim_{x\to 1^-}\dfrac{1}{1-x^2}$

   (b) $\lim_{x\to 1^+}\dfrac{x}{1-\sqrt{x}}$

Answers

1. (a) 2π
   (b) $\frac{4+\pi}{3}$

2. (a) 0
   (b) 179
   (c) $9 + 3\pi + \sqrt{2}$
   (d) 64
   (e) 5

3. (a) $\frac{1}{2}$
   (b) 0
   (c) $\sqrt{3}$
   (d) 0
   (e) $\frac{\pi}{4}$

4. (a) $\frac{3}{2}$
   (b) $2\sqrt{2}$
   (c) 2
   (d) $\frac{6}{5}$
   (e) -1
   (f) $\sqrt{2}$
   (g) 12.
   (h) $\frac{1}{6}$
   (i) 1
   (j) 0

7. $c = -2, d = 12$

8. (a) 3
   (c) Yes.

9. (a) 1
   (b) $\ln 0.025 \approx 7.3778$ hours

10. (a) $+\infty$
    (b) $-\infty$