Nanyang Technological University

SPMS/Division of Mathematical Sciences

2015/16 Semester 1 **MH1810 Mathematics I** **Tutorial 7**

Please be remined that there will be a 15-minute quiz during the tutorial session.

Reference Thomas' Calculus, Chapter 3.

1. Suppose f is differentiable and $f(x) > 0$.

   Use the following definition of derivative, $g'(x) = \lim_{t \to x} \dfrac{g(t) - g(x)}{t - x}$, to prove that

   (a) $\dfrac{d}{dx}(179f(x)) = 179f'(x)$.

   (b) $\dfrac{d}{dx}\sqrt{f(x)} = \dfrac{f'(x)}{2\sqrt{f(x)}}$.

   (c) $\dfrac{d}{dx}\left(\dfrac{1}{f(x)}\right) = \dfrac{-f'(x)}{(f(x))^2}$.

2. If $r(t) = \sin(f(t))$, $f(0) = \pi/3$, and $f'(0) = 4$, then what is $\dfrac{dr}{dt}$ at $t = 0$?

3. Calculate y'.

   (a) $y = \cos(\tan x)$

   (b) $y = \left(x + \dfrac{1}{x^2}\right)^{\sqrt{7}}$

   (c) $y = \dfrac{1}{\sin(x - \sin x)}$

   (d) $x^2 \cos y + \sin 2y = xy$

   (e) $x \tan y = y - 1$

   (f) $y = \ln(\sec x)$

   (g) $y = \ln(\sec x + \tan x)$

   (h) $y = \sin^{-1}(1 - x)$

4. Find the second derivative $f''(x)$ of $f(x) = \dfrac{x}{1 + x^2}$.

5. Find $f'(x)$.

   (a) $f(x) = \log_{10}\left(\dfrac{x}{x - 1}\right)$

   (b) $f(x) = \left(\dfrac{1 + \ln x}{1 - \ln x}\right)$

   (c) $f(x) = x \ln(1 + e^x)$

   (d) $f(x) = (\ln(1 + e^x))^2$

6. Find an equation of the tangent line to the curve $y = \dfrac{e^x}{x}$ at the point (i) $(1, e)$, (ii) where $x = -1$.

7. If n is a positive number, prove that

$$\frac{d}{dx}(\sin^n x \cos nx) = n \sin^{n-1} x \cos(n+1)x$$

8. (a) Use implicit differentiation to prove that

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}.$$

   (b) Use the formula established in part (a) to find $\dfrac{dy}{dx}$ for
   (i) $y = x\tan^{-1}\left(\frac{x}{2}\right)$, (ii) $y = \tan^{-1}(\ln x)$ and (iii) $\tan^{-1}(xy) = 1 + x^2 y$.

9. Find the derivative of the following function

$$f(x) = (\ln x)^{\cos x}, x > 1.$$

10. (Thomas' Calculus, Exercise 3.8, Q 16) The power P (watts) of an electric circuit is related to the circuit's resistance R (ohms) and current I (amperes) by the equation $P = RI^2$.

    (a) How are $\dfrac{dP}{dt}$, $\dfrac{dR}{dt}$ and $\dfrac{dI}{dt}$ related if P, R and I are functions of t?

    (b) How is $\dfrac{dR}{dt}$ related to $\dfrac{dI}{dt}$ if $P = P_0$ is constant?

11. (Thomas' Calculus, Exercise 7.7, Q 78 a) (**Accelerations whose magnitudes are propositional to displacement**) Suppose that the position of a body moving along a coordinate line at time t is $s = a\cos kt + b\sin kt$. Show that the acceleration $\dfrac{d^2 s}{dt^2}$ is proportional to s and it is directed to the origin.

12. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

13. A spotlight on the ground shines on a wall 12 m away. If a 2 m tall man walks from the spotlight straight towards the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing, at the moment when he is 4 m from the building?

14. (Thomas' Calculus, Exercise 3.8, Q36) A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (measured in meters) increases at a steady 10m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = 3$m?

Answers

3. (a) $-(\sec^2 x)\sin(\tan x)$
   (b) $\sqrt{7}(1-\frac{2}{x^3})(x+\frac{1}{x^2})^{\sqrt{7}-1}$
   (c) $\frac{-(\cos(x-\sin x))(1-\cos x)}{\sin^2(x-\sin x)}$
   (d) $\frac{2x\cos y-y}{x^2\sin y-2\cos(2y)+x}$
   (e) $\frac{\tan y}{1-x\sec^2 y}$
   (f) $\tan x$
   (g) $\sec x$
   (h) $\frac{-1}{\sqrt{1-(1-x)^2}}$

. . . .

4. $\frac{-2x(3-x^2)}{(1+x^2)^3}$

. . . .

5. (a) $\frac{-1}{(\ln\ 10)x(x-1)}$
   (b) $\frac{2}{x(1-\ln x)^2}$
   (c) $\ln(1+e^x)+\frac{xe^x}{1+e^x}$
   (d) $\frac{2e^x\ln(1+e^x)}{1+e^x}$

. . . .

6. (i) $y=e$

   (ii) $y=(-2/e)x-3/e$

. . . .

9. $(\ln x)^{\cos x}\left((-\sin x)\ln(\ln x)+\frac{\cos x}{x\ln x}\right)$

. . . .

10. (a) $\frac{dP}{dt}=I^2\frac{dR}{dt}+2RI\frac{dI}{dt}$
    (b) $\frac{dR}{dt}=-\frac{2R}{I}\frac{dI}{dt}=-\frac{2P_0}{I^3}\frac{dI}{dt}$

. . . .

12. $-\frac{1}{20\pi}\approx-0.0159$

. . . .

13. The shadow is getting shorter at a rate of 0.6 m/s.

. . . .

14. 1 rad/sec