Nanyang Technological University

SPMS/Division of Mathematical Sciences

2015/16 Semester 1 **MH1810 Mathematics I** **Tutorial 8**

Topics: Linearization, Differentials, Newton's Method, Closed Interval Method.

1. Use the result $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$ to determine the following limits.

   (a) $\lim\limits_{x\to 0}\dfrac{\sin x^2}{x^2}$

   (b) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}$

   (c) $\lim\limits_{x\to 0}\dfrac{\sin x^2}{\sin x}$

   (d) $\lim\limits_{x\to \pi}\dfrac{\sin(x-\pi)}{x-\pi}$

   (e) $\lim\limits_{x\to 0}\dfrac{\sin(\sin x)}{x}$

   (f) $\lim\limits_{x\to \infty} x\sin(\dfrac{1}{x})$

2. For the function $f(x)$, write down the linearization $L(x)=f(a)+(x-a)f'(a)$ at $x=a$.

   (a) $f(x)=\sqrt{x^2+9}$, $a=-4$

   (b) $f(x)=\tan x$, $a=\pi$

3. Use the linearization of a suitable function $f(x)$ at a point $x=a$ to approximate $\sqrt[3]{7.99}$.

4. The function $f(x)=2x^2+4x-3$ changes value when x changes from $x_0=-1$ to $x_1=-0.9$. Find

   (a) the change $\triangle f=f(x_1)-f(x_0)$;

   (b) the value of the estimate $df=f'(x_0)dx$, where $dx=x_1-x_0$; and

   (c) the approximation error $|\triangle f-df|$.

5. Write a differential formula that estimates the given change in the volume $V=\pi r^2h$ of a right circular cylinder when the height changes from h_0 to h_0+dh and the radius does not change.

6. Use Newton's method to estimate $\sqrt[4]{2}$, the positive fourth root of 2 by solving the equation $x^4-2=0$. Start with $x_0=1$ and find x_2.

7. Use the Intermediate Value Theorem to show that $f(x)=x^3+2x-4$ has a root in $(1,2)$. Then use Newton's Method to find the root to five decimal places. (Answer: 1.17951)

8. Use the closed interval method to find the global (absolute) maximum and minimum values of $f(x)=\sqrt{4-x^2}$ on the interval $[-2,1]$. (Answer: Global maximum: 2, global minimum: 0)

9. Find the global maximum and minimum values of $f(x)=e^x-2x$ on $[0,1]$.

10. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y=12-x^2$. What is the largest area the rectangle can have, and what are its dimensions?

Answers

1. (a) 1
   (b) 3
   (c) 0
   (d) 1
   (e) 1
   (f) 1

2. (a) $L(x) = 5 - \frac{4}{5}(x+4)$
   (b) $L(x) = (x - \pi)$

3. $\sqrt[3]{7.99} \approx 2 - \frac{1}{1200}$

4. (a) 0.02
   (b) 0
   (c) 0.02

5. $dV = (V'(h_0)) \cdot (dh) = \pi r^2 (dh)$

6. $x_2 = 1.1935$

7. Approximated Solution is $x = 1.179509$.

8. Global maximum: 2; Global minimum: 0

9. Global maximum is $f(0) = 1$; Global minimum is $f(\ln 2) = 2 - 2\ln 2$

10. The largest area is 32 and the rectangle has dimension 4 units by 8 units.