NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2012–2013

MH1810 – Mathematics I

MAY 2013 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **SEVEN (7)** printed pages, including Appendix.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT** an **OPEN BOOK** exam. However, a list of formulae is provided in the attachments.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(20 Marks)**

(a) Consider points $A(0,1,-1)$, $B(2,0,-1)$ and $C(3,-1,-2)$ in $\mathbb{R}^3$.

(i) Determine the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(ii) Find the area of the parallelogram whose edges are vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(iii) Determine the distance from the origin $(0,0,0)$ to the plane containing the above parallelogram.

(b) Consider the linear system

$$\begin{array}{rcrcrcl} x & + & 2y & + & 3z & = & b \\ 5x & + & y & + & z & = & 10 \\ 2x & & & - & z & = & 4 \end{array}.$$

Use Cramer's Rule to find the values of b such that $y \geq 0$.

QUESTION 2. **(20 Marks)**

(a) Determine whether each of the following limits exists. Justify your answer. If it exists, what is its value?

(i) $\displaystyle\lim_{x\to 1} \sqrt[3]{\frac{1}{\ln x} - \frac{1}{x-1}}$

(ii) $\displaystyle\lim_{x\to 0^+} (\sin x)\ln x$

(b) Find the value(s) of λ such that f is continuous at $x = 0$ where f is defined as follows.

$$f(x) = \begin{cases} \dfrac{\sin x + \lambda^2 e^{3x}}{5x - e^{3x}} & \text{if } -1 \leq x < 0, \\ \lambda \cos^{-1} x & \text{if } 0 \leq x \leq 1. \end{cases}$$

QUESTION 3. **(15 Marks)**

(a) Suppose $f(3) = \pi$ and $f'(3) = 5$.

Use the <u>definition of derivative</u> to prove that $g = \sqrt{4 + f(x)}$ is differentiable at $x = 3$, and determine $g'(3)$.

(b) Find the following derivative

$$\frac{d}{dx}\left(\int_3^{\sqrt{5x^2+\sin x}} \cos(t^4)\, dt\right).$$

QUESTION 4 **(15 Marks)**

(a) Use the linearization $L(x)$ of a suitable function $f(x)$ at $x = a$ to approximate the value $\sqrt[3]{0.124}$.

(b) Find the exact volume of the solid when the region bounded by the curve $y = \ln(x + 1)$ and lines $x = 0, x = 3$ and $y = 0$ is rotated about the y-axis.

QUESTION 5 **(15 Marks)**

(a) Determine the indefinite integral $\displaystyle\int \frac{x}{x^2 + 6x + 13}\, dx$

(b) Express the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k \sin\left(\frac{k\pi}{n}\right)}{n^2}$$

as a definite integral $\displaystyle\int_0^1 f(x)\, dx$ for some function f and hence evaluate its value.

QUESTION 6 **(15 Marks)**

A rectangle has its base on the x-axis and its upper two vertices on the curve $y = \cos x$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(a) Express the area $A(x)$ of the rectangle in terms of $x \in \left(0, \frac{\pi}{2}\right)$.

(b) Explain why $A'(x) = 0$ has exactly one solution in the interval $\left(0, \frac{\pi}{2}\right)$.

(c) Use Newton's Method (2 iterations) to estimate the value $x^* \in \left(0, \frac{\pi}{2}\right)$ where the area $A(x^*)$ is maximum. (You may express x^* up to 4 decimal places.)

END OF PAPER

Appendix

Numerical Methods.

- Linearization Formula:

$$L(x) = f(a) + f'(a)\,(x-a)$$

- Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[y_0 + 2\,(y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$$

- Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right]$$

Derivatives.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \text{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\text{csch}^2 x$$

$$\frac{d}{dx}(\text{sech } x) = -\text{sech} x \tanh x \qquad \frac{d}{dx}(\text{csch } x) = -\text{csch} x \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

Antiderivatives.

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1 \qquad \int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \sin x \, dx = -\cos x + C \qquad \int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C \qquad \int \csc^2 x \, dx = -\cot x + C$$

$$\int \tan x \sec x \, dx = \sec x + C \qquad \int \cot x \csc x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C \qquad \int \cot x \, dx = \ln|\sin x| + C$$

$$\int e^x \, dx = e^x + C \qquad \int a^x \, dx = \frac{a^x}{\ln a} + C,\ a > 0$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C,\ |x| < |a| \qquad \int \frac{1}{x^2+a^2} \, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1}|x| + C,\ |x| > 1 \qquad \int \frac{1}{\sqrt{x^2+1}} \, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{x\sqrt{x^2-a^2}} \, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C,\ |x| > |a| \qquad \int \frac{1}{\sqrt{x^2+a^2}} \, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

MH1810 MATHEMATICS 1

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.