NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2013–2014
MH1810 – Mathematics I

NOVEMBER 2013 TIME ALLOWED: 2 HOURS

Matriculation Number: | | | | | | | | | |

Seat Number: | | | | | |

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **NINETEEN (19)** pages, including an Appendix.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. This **IS NOT** an **OPEN BOOK** exam. However, a list of formulae is provided in the Appendix.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

For examiners only.

Questions	Marks
1 (20)	
2 (10)	
3 (20)	

Questions	Marks
4 (20)	
5 (15)	
6 (15)	

Total (100)	

QUESTION 1. **(20 Marks)**

(a) Let r be a real constant. Consider the linear system

$$\begin{array}{rcrcrcr} & & ry & + & z & = & r \\ -x & & & + & z & = & 0 \\ x & + & 3y & + & (r+1)z & = & -r \end{array}$$

(i) Let $A = \begin{pmatrix} 0 & r & 1 \\ -1 & 0 & 1 \\ 1 & 3 & (r+1) \end{pmatrix}$. Evaluate $\det(A)$ by cofactor expansion along the third column.

(ii) State condition(s) on r for Cramer's Rule to be applicable to solve the above linear system.

(iii) Use Cramer's Rule to find x in terms of r.

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(b) Solve the equation

$$(1-\sqrt{3}i)z^3 = -4+4i.$$

Express z as $r(\cos q\pi + i\sin q\pi)$, where $r > 0$, q is a rational number and $q \in (-1, 1]$.

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QUESTION 2. **(10 Marks)**

Consider the function $f : \mathbb{R} \to \mathbb{R}$ where

$$f(x) = \begin{cases} (\sin x^2)\left(\cos(\frac{1}{x^2})\right) & \text{if } x < 0, \\ \dfrac{\sqrt{4x^2+9}-3}{3x+2+\sin x} & \text{if } x \geq 0. \end{cases}$$

(a) Does $\lim\limits_{x \to 0^+} f(x)$ exist? If it does, what is its value? If the limit does not exist, explain why.

(b) Is f continuous at $x = 0$? Justify your answer.

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QUESTION 3. **(20 Marks)**

Let $f(x) = 4x^3 - 9x^2 - 12x + 5$ on $[-1, 3]$.

(a) Use the closed interval method to find the range of f on $[-1, 3]$.

(b) Use Intermediate Value Theorem to explain why the graph of $y = f(x)$ cuts the x-axis at $x = c$ for some $c \in (0, 1)$.

(c) Use Newton's Method, with $x_0 = 0$, to approximate c in part(b) by the first iterate x_1.

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QUESTION 4. **(20 Marks)**

(a) Use the linearization $L(x)$ of

$$g(x) = 3 + x \int_1^{x^2} \sec(t-1)dt$$

at $x = 1$ to approximate the value $g(1.01)$.

(b) A bowl has a shape that can be generated by revolving the graph of $y = x^2/2$ between $y = 0$ and $y = 5$ about the y-axis.

If the bowl is filled with water at a constant rate of 3 cubic units per second, how fast will the water level be rising when the water is 4 units deep?

QUESTION 5. **(15 Marks)**

The base of a solid is the region R bounded by the graphs of $y = \sqrt{x}$ and $y = x/2$. The cross sections perpendicular to the x-axis are squares whose sides run across the base of the solid.

(a) Find the area of the bounded region R.

(b) Find the volume of the solid.

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QUESTION 6. **(15 Marks)**

(a) Let $I_n = \int_0^t x^n e^{-3x}\, dx$, for $n = 0, 1, 2, \ldots$.

Prove the following reduction formula for I_n,

$$I_n = \frac{1}{3}\left(-t^n e^{-3t} + nI_{n-1}\right), n \geq 1.$$

(b) Evaluate the improper integral $\int_0^\infty x^2 e^{-3x} dx$.

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END OF PAPER

Appendix

Numerical Methods.

- Linearization Formula:

$$L(x) = f(a) + f'(a)\,(x - a)$$

- Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[y_0 + 2\left(y_1 + y_2 + \cdots + y_{n-1}\right) + y_n\right]$$

- Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right]$$

Derivatives.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x \qquad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

Antiderivatives.

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1$$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \tan x \sec x \, dx = \sec x + C$$

$$\int \cot x \csc x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \cot x \, dx = \ln|\sin x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C, \; a > 0$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$$

$$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \; |x| < |a|$$

$$\int \frac{1}{x^2+a^2} \, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1}|x| + C, \; |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}} \, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{x\sqrt{x^2-a^2}} \, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C, \; |x| > |a|$$

$$\int \frac{1}{\sqrt{x^2+a^2}} \, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

MH1810 MATHEMATICS 1

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.