NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2013–2014
MH1810 – Mathematics 1

MAY 2014 TIME ALLOWED: 2 HOURS

Matriculation Number:

Seat Number:

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **SIXTEEN (16)** pages, including an Appendix.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. This **IS NOT** an **OPEN BOOK** exam. However, a list of formulae is provided in the attachments.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

For examiners only

Questions	Marks
1 (15)	
2 (15)	
3 (20)	

Questions	Marks
4 (15)	
5 (15)	
6 (20)	

Total (100)	

QUESTION 1. **(15 Marks)**

(a) Let $z = 1 - i\sqrt{3}$. Determine the following complex numbers and find the polar representation of all numbers and plot them on the complex plane:

$$iz^{-2},\ (1+i)z^3,\ \frac{z}{\bar{z}},\ z^2+4$$

Question 1 continues on the next page

(b) Consider vectors $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

(i) Find a unit vector that is perpendicular to vectors $\mathbf{u}$ and $\mathbf{v}$.

(ii) Determine the scalar equation of the plane Π which passes through the point $(1, 1, 0)$ and is parallel to $\mathbf{u}$ and $\mathbf{v}$. What is the distance between plane Π and the plane containing $\mathbf{u}$ and $\mathbf{v}$?

QUESTION 2. **(15 Marks)**

(a) Find the limit:

$$\lim_{x \to 3} \frac{\sqrt{x+13} - 2\sqrt{x+1}}{x^2 - 9}$$

Question 2 continues on the next page

(b) Consider the function f which is defined as follows:

$$f(x) = \begin{cases} \dfrac{e^x}{2+x} & \text{if } x \leq 0, \\ \cos(1-e^{\pi x}) & \text{if } x > 0. \end{cases}$$

Is f differentiable at $x = 0$? Justify your answer.

QUESTION 3. **(20 Marks)**

(a) Used the closed interval method to find the absolute minimum and absolute maximum of the function $f(x) = (x-1)^2 + x$ on the interval $[0, 1]$.

Question 3 continues on the next page

(b) Use the intermediate value theorem to show that the function $f(x) = \cos(x) - x$ has a root between 0 and π.

Question 3 continues on the next page

(c) Let $a, b,$ and c be real numbers. Use the mean value theorem to show that the equation

$$4ax^3 + 3bx^2 + 2cx = a + b + c$$

always has a root between 0 and 1.

QUESTION 4. **(15 Marks)**

Consider the function f defined as follows: $f(x) = (x+6)^3(x-2)$.

(a) Find the intervals of monotonicity of the function.

(b) Find the intervals of concavity/convexity of the function f and the possible inflection points.

QUESTION 5. **(15 Marks)**

(a) Find the curve $y = f(x)$ that passes through the point $(4, 9)$ and whose gradient at each point (x, y) is $2\sqrt{x}$.

Question 5 continues on the next page

(b) Find the volume of the solid obtained by revolving the region bounded by the curves $y = x$ and $y = \sqrt{x}$ about the line $x = 2$.

QUESTION 6. **(20 Marks)**

(a) Evaluate the definite integrals: $\displaystyle\int_{-\pi}^{\pi/2} f(x)\,dx$,

where

$$f(x) = \begin{cases} e^x & \text{if } -\pi \le x \le 0, \\ \cos x & \text{if } 0 < x \le \pi. \end{cases}$$

Question 6 continues on the next page

(b) When the region bounded by the x-axis and the curve $y = \sqrt{r^2 - x^2}$ for $-r \leq x \leq r$ is rotated about the x-axis, we get a sphere with radius r. Find the volume of the sphere.

END OF PAPER

Appendix

Numerical Methods.

- Linearization Formula:

$$L(x) = f(a) + f'(a)\,(x - a)$$

- Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[y_0 + 2\,(y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$$

- Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right]$$

Derivatives.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \text{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\text{csch}^2 x$$

$$\frac{d}{dx}(\text{sech } x) = -\text{sech} x \tanh x \qquad \frac{d}{dx}(\text{csch } x) = -\text{csch} x \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

Antiderivatives.

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1 \qquad \int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \sin x \, dx = -\cos x + C \qquad \int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C \qquad \int \csc^2 x \, dx = -\cot x + C$$

$$\int \tan x \sec x \, dx = \sec x + C \qquad \int \cot x \csc x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C \qquad \int \cot x \, dx = \ln|\sin x| + C$$

$$\int e^x \, dx = e^x + C \qquad \int a^x \, dx = \frac{a^x}{\ln a} + C, \; a > 0$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \; |x| < |a| \qquad \int \frac{1}{x^2+a^2} \, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1}|x| + C, \; |x| > 1 \qquad \int \frac{1}{\sqrt{x^2+1}} \, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{x\sqrt{x^2-a^2}} \, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C, \; |x| > |a| \qquad \int \frac{1}{\sqrt{x^2+a^2}} \, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

MH1810 MATHEMATICS 1

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.