NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2014-2015

MH1810 – MATHEMATICS 1

April 2015 Time Allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. Nevertheless, they should write down systematically the steps in their workings.

QUESTION 1. **(20 marks)**

Consider the function f defined by

$$f(x) = \begin{cases} \dfrac{10\cos\sqrt{2-x}}{x^2+1} & \text{when } x < 2, \\ 1 & \text{when } x = 2, \\ \sqrt{x^2+6x} - x & \text{when } x > 2. \end{cases}$$

Determine whether each of the following limits exists. If so, find its value. If not, give a brief justification.

(a)
$$\lim_{x\to-\infty} f(x)$$

(b)
$$\lim_{x\to 2} f(x)$$

(c)
$$\lim_{x\to\infty} f(x)$$

QUESTION 2. **(14 marks)**

Consider a cone of height H and base diameter D.

(a) Use linearization to estimate the allowable percentage error in the measurement of D if the volume of the cone is to be determined to within 2% of its true value.

(b) Assuming that the total surface area of the cone equals π, find the value of D for which the cone has maximum volume.
Hint: Recall that the total surface area of a cone is a sum of the area of the base and the lateral area. Use the formula $A_{\text{Lateral}} = \pi RL$ for the lateral area of the cone, where R is the base radius and L is the lateral height.

QUESTION 3. **(25 marks)**

(a) Find the following derivative:

$$\frac{d}{dx}\int_{3\sin x}^{2015} t^2 \cos t \, dt.$$

(b) Find the value of the integral

$$\int_0^{0.5} \frac{11x+5}{x^2-x-12}\, dx.$$

(c) The region bounded by the graph of the function

$$f(x) = \sqrt{x}e^{-x^2}$$

and by the x-axis is rotated about the x-axis. Calculate the exact volume of the resulting solid.

QUESTION 4. **(16 marks)**

Consider the plane Π passing through the points $A = (1, 1, 0)$, $B = (-2, 0, 1)$, and $C = (0, -1, -1)$.

(a) Find an equation describing Π.

(b) Determine the angle that Π makes with the plane $z = 0$.

(c) Find an equation of the line passing through the point A and perpendicular to the plane Π.

QUESTION 5. **(5 marks)**

Find conditions that the parameters a and b have to satisfy in order for the matrix

$$\begin{pmatrix} a & 0 & b \\ 0 & 1 & 0 \\ b & 0 & a \end{pmatrix}$$

to have an inverse.

QUESTION 6. **(20 marks)**

(a) Find all complex fourth roots of -81.
Hint: Solve the equation $z^4 = -81$ in complex numbers.

(b) Find all complex numbers z for which $\bar{z} = z^2$.

(c) Express the complex number

$$\frac{\frac{27}{16}\left(i\sqrt{3}-1\right)^3 - \frac{1}{2}}{2+3i}$$

in the form $x + iy$ with x and y real.

END OF PAPER

MH1810 MATHEMATICS 1

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.